

# Non-perturbative BRST invariance and what it might be good for

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We construct a local, gauge-fixed, lattice Yang-Mills theory with an exact BRST invariance, and with the same perturbative expansion as the standard Yang-Mills theory. The ghost sector, and some of its BRST transformation rules, are modified to get around Neuberger's theorem. A special term is introduced in the action to regularize the Gribov horizons, and the limit where the regulator is removed is discussed. We conclude with a few comments on what might be the physical significance of this theory. We speculate that there may exist new strong-interaction phases apart from the anticipated confinement phase. (For additional technical details see ref. [1].)

## 1. Introduction

The conventional lattice formulation of gauge theories does not require gauge fixing because the link variables take values in the compact gauge group. At the same time, we have gained a lot of knowledge about gauge theories from perturbation theory, where gauge fixing is indispensable. This includes for example the celebrated asymptotic freedom of non-abelian gauge theories. Given these facts, it is not unlikely that we could learn new things if we had at our disposal a local, non-perturbative (*i.e.* lattice) formulation that makes closer contact with standard perturbation theory, *including* its gauge-fixing sector.

Most common in perturbation theory are covariant gauges for which power-counting renormalizability is manifest. Here we will focus on pure Yang-Mills (YM) theory with the covariant gauge-fixing term  $(\partial_\mu A_\mu)^2/(2\xi g^2)$ , where  $g$  is the gauge coupling and  $\xi$  is the gauge parameter. The gauge-fixed action contains also the ghost term  $\bar{C}_a \Omega_{ab} C_b$  where the (real, but in general non-hermitian) Faddeev-Popov operator is  $\Omega_{ab} = \partial_\mu D_{ab\mu}$ , and  $D_{ab\mu} = \delta_{ab} \partial_\mu + f_{acb} A_{c\mu}$ . The gauge-fixed action is invariant under the BRST transformation. The latter resembles a gauge transformation with a grassmannian parameter, the ghost field  $C(x)$ , and plays a key role in proving renormalizability and unitarity to all orders.

We may now state our goal by asking the following question: *can one construct a local,*

*gauge-fixed, BRST-invariant lattice formulation of Yang-Mills theory?* Success is by no means guaranteed, but there are good reasons to try. Among our motivations we list the following.

- 1) A non-perturbative construction of *non-abelian chiral gauge theories* is another long-standing open problem. In its absence, our knowledge of the dynamics of chiral gauge theories lags way behind our understanding of QCD. It has been demonstrated convincingly that fermions with chiral coupling to an (abelian) gauge field exist on the lattice if the action contains a covariant gauge-fixing term [2]. Gauge fixing may thus prove to be a key element in the lattice construction of non-abelian chiral gauge theories as well.
- 2) A BRST-invariant gauge-fixed lattice theory might, possibly, contain new phases except the familiar confinement phase (see section 5).

## 2. The Gribov problem

By definition, a gauge-fixing term destroys gauge invariance of the Boltzmann weight  $\exp(-S)$ , where  $S$  is the action. Gauge-invariant observables will remain intact provided the following integral over any gauge orbit

$$\int \mathcal{D}\phi \mathcal{D}(\text{ghosts}) \exp(-S(A_\mu^\phi, \text{ghosts})), \quad (1)$$

is a non-zero constant. Here  $\phi(x)$  parameterizes a gauge transformation and  $A_\mu^\phi$  is the rotated field.

In perturbation theory, which is a saddle-point expansion around the classical vacuum  $A_\mu = 0$ , this condition is satisfied. Non-perturbatively, this is not necessarily true, because of the existence of Gribov copies, namely, multiple solutions of the gauge condition  $\partial_\mu A_\mu^\phi = 0$  on the same orbit [3]. In this situation the correct condition is that  $\sum \text{sign}(\det(\Omega))$  must be a non-zero (integer) constant, where the sum is over all Gribov copies on a given orbit. Geometrically, one expects that that constant is the index of some mapping [4].

In order to test this condition in a well-defined, non-perturbative setting, one must resort to the lattice. Doing so, a no-go theorem was discovered by Neuberger [5]. Considering a general class of BRST-invariant lattice theories, he proved, under certain assumptions, that the orbit integral (1) is indeed a constant, but this constant is equal to zero! Consequently the partition function itself, as well as unnormalized expectation values of gauge-invariant operators, vanish.

Ways around Neuberger's theorem have been found. The trick of ref. [6] is special to a  $U(1)$  gauge group, and is therefore by itself of limited usefulness. Another approach, where a non-abelian group is partially gauge-fixed to a maximal abelian sub-group, was devised in ref. [7]. (It may be interesting to combine these two methods into one gauge-fixing scheme.)

### 3. Modified BRST transformations

Here we will circumvent Neuberger's theorem by modifying the ghost sector such that, when acting on the new ghost-sector fields, BRST transformations cease to be nilpotent. We are also guided by the consideration that we would like to have a *non-negative* ghost-sector partition function, because positivity of the measure is crucial for using existing numerical techniques.

Our starting point is the off-shell form of the Faddeev-Popov gauge-fixing action

$$S_{\text{FP}} = \frac{\tilde{g}^2}{2} \lambda^2 + i\lambda \partial_\mu A_\mu + \overline{C} \Omega C, \quad (2)$$

where  $\lambda$  is the auxiliary field and  $\tilde{g}^2 = \xi g^2$ . (We use continuum notation for simplicity, but everything can be done on the lattice in terms of the fa-

miliar compact link variables for the gauge field.) We first perform a change of variables in the ghost sector. Instead of the  $\overline{C}$  field, we introduce a new grassmann field  $\chi$  and a real scalar field  $\eta$ , both carrying a (suppressed) adjoint index. At this point the gauge-fixing action is taken to be

$$S_{\text{gf}} = \frac{\tilde{g}^2}{2} \lambda^2 + i\lambda \partial_\mu A_\mu + \chi^T \Omega^T \Omega C + S_1, \quad (3)$$

where

$$S_1 = \frac{h^2}{2} \left( \hat{\delta}(\Omega\chi) + i\lambda \right)^2 \quad (4)$$

$$= \frac{1}{2} \left( \Omega\eta + h(\hat{\delta}\Omega\chi + i\lambda) \right)^2. \quad (5)$$

Here  $h$  is a new coupling constant.  $\hat{\delta}$  is the BRST variation which, for the new field  $\chi$ , is chosen to be  $\hat{\delta}\chi(x) = \eta(x)/h$ .

A comparison of the old (eq. (2)) and new (eq. (3)) gauge-fixing actions reveals that, in effect, we have made the non-linear change of variables  $\overline{C} \rightarrow (\Omega\chi)^T$ . The necessary jacobian is provided by the integral over the new  $\eta$  field. Indeed, the gaussian  $\eta$ -integration “shifts away” the  $h$ -dependent term in eq. (5), resulting in a factor of  $|\det(\Omega)|^{-1}$ . (Thus  $h$  drops out of the perturbative expansion.) The grassmann-ghosts integral now yields  $\det^2(\Omega)$ . Putting these together we obtain (formally!) that the ghost-sector partition function is  $|\det(\Omega)|$ . Needless to say, perturbation theory is unchanged, because in this context the sign of the determinant is inconsequential.

The familiar off-shell  $\overline{C}$  transformation rule is  $\hat{\delta}\overline{C} = -i\lambda$ . For the new action, this relation is recovered as the global minimum of the  $\eta$ -action  $S_1$ ,  $\hat{\delta}(\Omega\chi) = -i\lambda$ . Moreover, the new action is BRST invariant provided we choose  $\hat{\delta}\eta(x) = -C(x)/h$ . (For this to be true, the  $h$ -dependent terms in eq. (5) are essential.) Nilpotency is lost because  $\hat{\delta}^2\chi(x) = -C(x)/h^2 \neq 0$ . In a non-abelian theory  $\hat{\delta}^2\eta(x) \neq 0$  too, while  $\hat{\delta}^2$  still vanishes on all other fields. Note that the mass dimension of all the ghost-sector fields is now zero.

### 4. Gribov-horizon regulator

In perturbation theory, the gauge-fixing actions (2) and (3) yield identical expressions for

all correlation functions with no external ghost legs. Non-perturbatively, however, the  $\eta$ -integral is ill-defined on the Gribov horizons, where the Faddeev-Popov operator has zero modes. To tame this singularity we add a new *horizon-regulator* term to the action,  $m^2 S_2$ , where

$$S_2 = \chi^T D_\mu^T D_\mu C + \frac{1}{2} \left( D_\mu \eta + h \hat{\delta}(\text{Ad} A_\mu) \chi \right)^2 \quad (6)$$

and  $((\text{Ad} A_\mu) \chi)_a = f_{abc} A_{b\mu} \chi_c$ . The new addition (6) to the action is BRST invariant as well. For certain (*e.g.* Schrödinger functional) boundary conditions, it can be shown that the lattice  $\eta$ -action has no zero modes, if  $m \neq 0$ . The lattice theory is therefore well-defined, and BRST invariant. The target theory is defined by taking both the continuum limit and the  $m \rightarrow 0$  limit.

## 5. Outlook

We have constructed a new non-perturbative formulation of YM theory. This gauge-fixed lattice theory is not necessarily in the same universality class as the familiar lattice YM. In perturbation theory, though, we can set  $m = 0$ , and so the two theories share an identical set of gauge-invariant correlation functions to all orders. Proving unitarity of the gauge-fixed theory beyond perturbation theory is an open question.

Non-perturbatively, the gauge-fixed partition function receives new contributions for  $m \rightarrow 0$ . They are  $\delta$ -function-like distributions localized on the Gribov horizons and proportional to  $h$  (cf. eqs. (5,6); we have verified this in a toy model [1]). If we send  $h \rightarrow 0$ , these distributions vanish and the ghost-sector partition function reduces to  $|\det(\Omega)|$  exactly. The double limit  $m, h \rightarrow 0$  may thus provide a starting point for numerical investigations using existing techniques.

The symmetries of the gauge-fixed theory include BRST and a global remnant,  $G$ , of the local gauge group. It will be interesting to explore the phase diagram and study the realization of these symmetries in each phase. An intriguing fact is that the dynamics of the longitudinal degrees of freedom is controlled by the coupling constant  $\tilde{g}$  which is *asymptotically free*, too (cf. eq. (2)); this result follows from the known

$\beta$ -functions of the gauge coupling and the gauge parameter [8]. As a result, the gauge-fixed theory has *two* dynamically-generated infra-red scales  $\Lambda$  and  $\tilde{\Lambda}$ , associated respectively with the renormalized coupling constants  $g$  and  $\tilde{g}$ .

For  $\Lambda \gg \tilde{\Lambda}$  we anticipate the existence of a phase with unbroken symmetries, that may be identified with the familiar confinement phase. We speculate that, for  $\tilde{\Lambda} \gg \Lambda$ , there may exist a new phase where some of the global symmetries are broken spontaneously.

Of particular interest would be a phase where the global symmetry  $G$  is broken spontaneously. Physically, such a phase would represent a new, *dynamical* Higgs mechanism, where (some of) the gauge bosons acquire a mass of order  $\tilde{\Lambda}$ . A necessary (and sufficient?) condition for the consistency of this phase is that its low-energy description is given by a conventional, renormalizable Higgs lagrangian. (Thus, our scenario might provide a new solution to the Higgs-triviality problem.) In the present context the effective Higgs field should be a bound state made of glue, so it must be an adjoint Higgs. Work on these and other issues is in progress.

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## REFERENCES

1. <http://www.physics.sfsu.edu/~maarten/>
2. For a recent review on the gauge-fixing approach see: M. Golterman and Y. Shamir, hep-th/0202162, and references therein; For a general review (LATTICE 2000, plenary) see: M. Golterman, hep-lat/0011027
3. V.N. Gribov, Nucl. Phys. **B139** (78) 1; I.M. Singer, Comm. Math. Phys. **60** (78) 7
4. P. Hirschfeld, Nucl. Phys. **B157** (79) 37
5. H. Neuberger, Phys. Lett. **B175** (86) 69
6. M. Testa, Phys. Lett. **B429** (98) 349
7. M. Schaden, Phys. Rev. **D59** (98) 014508
8. D. Espriu, P. Pascual and R. Tarrach, J.Phys. **G7** (81) 1591; see also H. Hata, Phys. Lett. **B143** (84) 171